DOCUMENT RESUME

ED 356 234 TM 019 643

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TITLE An Explanation of Canonical Correlation Analysis as the Most General Linear Model with Heuristic Examples.
PUB DATE Jan 93
NOTE 28p.; Paper presented at the Annual Meeting of the Southwest Educational Research Association (Austin, TX, January 28-30, 1993).
PUB TYPE Reports - Evaluative/Feasibility (142) -- Speeches/Conference Papers (150)

EDRS PRICE MF01/PC02 Plus Postage.
DESCRIPTORS *Analysis of Variance; *Correlation; *Discriminant Analysis; *Heuristics; *Multivariate Analysis; *Regression (Statistics)
IDENTIFIERS *Linear Models; Parametric Analysis; Pearson Product Moment Correlation; T Test

ABSTRACT

Using a hypothetical data set of 24 cases concerning opinions on contemporary issues on which Democrats and Republicans might disagree, concrete examples are provided to illustrate that canonical correlation analysis is the most general linear model, subsuming other parametric procedures as special cases. Specific statistical techniques included in the analysis are "t"-tests, Pearson correlation, multiple regression, analysis of variance, multiple analysis of variance, and discriminant analysis. The discussion is aided by an initial explanation of the logic of canonical analysis. The equivalence of results, common features of the statistical methods, and the superiority of canonical methods are highlighted. Similarities between the canonical technique and other univariate and multivariate procedures are emphasized. Twelve tables present analysis results. An appendix lists the Statistical Analysis System commands for all parametric tests. (Contains 22 references.) (Author/SLD)

An Explanation of Canonical Correlation Analysis as the Most General Linear Model with Heuristic Fxamples

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Running head: Canonical Subsumes All

A paper presented at the annual meeting of the Southwest Educational Research Association, Austin, TX, January 29, 1993.

Abstract

Using a hypothetical data set, concrete examples are provided to illustrate that canonical correlation analysis is the most general linear model, subsuming other parametric procedures as special cases. Specific statistical techniques included in the analysis are *t*-tests, Pearson correlation, multiple regression, ANOVA, MANOVA, and discriminant analysis. The discussion is aided by an initial explanation of the logic of canonical analysis. Further, similarities between the canonical technique and other univariate and multivariate procedures are highlighted.

Although multivariate statistical methodology has existed for many years, its use by behavioral scientists has been limited because of the difficulty of performing the analyses and the complexity of interpreting the results. This state of affairs has continued despite the widespread availability of computerized statistical packages which simplify the execution of such procedures (LeCluyse, 1990). Fish (1988) suggests that a lack of understanding of multivariate techniques is a possible cause for their limited usage. Given the complex interrelationships that exist among multiple variables in education and other social science settings, it is important that researchers become conversant with multivariate analytic techniques.

Univariate methods can be used to test hypotheses about the effect of several independent (predictor) variables on a single dependent (criterion) variable. Multivariate methods, on the other hand, examine not only a set of independent variables, but also a set of two or more dependent variables simultaneously. This attribute makes them vital in behavioral studies (Fish, 1988, p.130; Thompson, 1986b). As Campbell (1992) explains, "because human behavior includes multiple facets, each of which is affected by a wide range of variables, many behavioral studies ask questions that involve both multiple independent and multiple dependent variables" (p. 1). In such cases, multivariate methods should be employed.

Fish (1988) discusses three reasons why multivariate techniques are so important in behavioral research. The first is

-that multivariate statistics limit the experimentwise Type I error rate. Researchers often set stringent alpha levels in order to avoid a *testwise* Type I error (erroneously rejecting a null hypothesis). Unfortunately, when several hypotheses regarding the same data set are tested using multiple univariate tests, the *experimentwise* alpha level is actually inflated to an unknown degree, leaving the researcher uncertain as to which "statistically significant results are errors and which are not" (Thompson, 1991, p. 80). Multivariate methods circumvent this problem. By simultaneously testing relationships among all of the variables, the possibility of a Type I error in the experiment is limited to alpha (e.g., .05, .01, etc.).

The second reason multivariate techniques should be more widely used is to increase power against committing a Type II error (failing to detect statistically significant results). Thompson (1988) explains that the "failure to employ multivariate methods can lead to a failure to identify statistically significant results which actually exist" (p. 12). As Campbell (1992) notes, "although statistical significance is mainly influenced by sample size and is not the *raison d'etre* of research, it is still the benchmark by which many journal editors guide their publication decisions" (p.2). Hence, by choosing a univariate rather than a multivariate method, a researcher may not only be choosing an analytic technique that masks the hoped for effect, but may thereby lose an opportunity to publish.

The third and most important reason for using multivariate techniques is that they "best honor the reality to which the researcher is purportedly trying to generalize" (Thompson, 1988, p. 12). This reality is human behavior, which involves multiple causes and multiple effects. McMillan and Schumacher (1989) assert that "in the reality of complex social situations the researcher needs to examine many variables simultaneously" (p. 207). Such analyses are possible only with multivariate techniques which enable us to understand the relationships among several variables at one time.

Kerlinger (1986) contends that the "preoccupation of behavioral scientists today is more likely to be with multiple relations" (p. 65), making the use of multivariate procedures essential. According to Campo (1990), "multivariate statistical methods deal with the full system of interrelationships between variables and thus shed the most light upon how variables work together and influence each other" (p. 1). Canonical correlation analysis is the most powerful multivariate technique for studying such interrelationships.

Although analysis of variance (ANOVA) techniques have been used extensively, Thompson (1986a) shows general linear models, such as multiple regression and canonical correlation analysis, to be superior. Since Cohen's (1968) classic article on the use of regression as the most general linear model, subsuming analyses of variance techniques, the use of multiple regression dramatically increased (Willson, 1982). However, as Knapp (1978) demonstrates, canonical correlation analysis, and not regression, is the most general case of the general linear model. Thus, "virtually all of the commonly encountered parametric tests of significance can be treated as special cases of canonical correlation analysis, which is the general procedure for investigating the relationships between two sets of variables" (Knapp, 1978, p. 410). Fornell (1978) makes this point more specifically stating that "multiple regression, MANOVA and ANOVA, and multiple discriminant analysis can all be shown to be special cases of canonical analysis. Principal components analysis is also in the inner family circle" (p. 168).

Thompson (1984) points out, however, that while the recognition of analyses of variance methods as special cases of regression generated increased usage of multiple regression, the recognition of canonical correlation analysis as the most general case of the general linear model did not produce a similar increase in the use of canonical techniques. Notwithstanding the increased use of regression analysis, researchers (Edgington, 1974; Willson, 1980; Goodwin & Goodwin, 1985) report that most studies published in prominent research journals over the last several decades have employed analysis of variance techniques (ANOVA, ANCOVA, MANOVA, MANCOVA).

The purpose of the current paper is to explain the logic of canonical correlation analysis and to demonstrate, with the use of a hypothetical data set, that canonical correlation subsumes other parametric significance tests as special cases. Six familiar univariate and multivariate procedures are demonstrated in a series of paired calculations that includes the results of each procedure and then compares the results to canonical results using the same variables. The procedures examined include t-test, Pearson correlation, multiple regression, ANOVA, MANOVA, and discriminant analysis. The equivalence of results and the common features of the statistical methods are highlighted, and the superiority of canonical methods is examined. The discussion also includes the logic of canonical analysis and an explanation of the equivalence of the standardized weights for the various procedures.

A Discussion of Canonical Correlation Analysis

Canonical correlation analysis is a multivariate technique used to study relationships between two variable sets, a predictor set and a criterion set. For canonical analysis, each set has at least two variables. As Thompson (1984) indicates, canonical correlation analysis can be presented in bivariate terms, which facilitates the explanation since most researchers feel comfortable with the bivariate correlation coefficient. As a matter of fact, canonical correlation coefficient is simply the Pearson correlation between scores in the two variable sets.

Similar to other procedures, synthetic scores must be developed before the canonical correlation can be calculated. These synthetic scores are similar to YHAT scores (the predicted score) in regression analysis, to factor scores in factor analysis, and to discriminant scores in discriminant function analysis. As with other parametric methods, the synthetic scores are the focus of the analysis in canonical correlation (Thompson, 1988).

The synthetic scores are produced the same way across statistical procedures. A weight for each variable is developed "to optimize some condition" (Thompson, 1991, p. 81). These weights are equivalent, though terminology differs across procedures. According to Taylor (1992), they are called beta weights in regression, factor pattern coefficients in factor analysis, discriminant function coefficients in discriminant analysis, canonical function coefficients in canonical correlation analysis. In addition to weights, the original scores on each variable are converted to *z*-scores. Then, for each subject, and on each variable, the *z*-scores are multiplied by weights and the resulting values are summed across each variable in the set. Taylor (1992) notes that these "weights are applied to...the predictor and criterion variables in such a way that redundancy among the synthetic composites is eliminated" (p. 70). The result is a composite set of synthetic predictor scores and a composite set of synthetic criterion scores, scores on which the canonical analysis is based.

In canonical correlation, functions similar to principal components are derived from the synthetic scores; each function represents a linear combination of both the predictor and criterion composite scores, and is orthogonal, or uncorrelated, with the other functions. The number of functions extracted in canonical analysis equals the number of variables in the smaller of the two variable sets (Thompson, 1991). Thus, the first function extracted accounts for the maximum possible correlation between the two sets of scores and produces the largest canonical correlation (*Rc*). The second function represents the maximum correlation possible from the remaining variance, subject to the restriction that it be perfectly uncorrelated with the first function (Campo, 1990). Each successive function is extracted in this manner.

Viewing analytical techniques such as canonical correlation as measures of bivariate relationships among variables enables one to understand the reasoning behind canonical correlation analysis and why it produces the same research results as other parametric methods. Kerlinger (1986) contends that the study of relationships among variables is the basis of scientific research, and that there is no empirical way to know anything except through its relations to other things. Thompson (1991) asserts further that "*all* classical *analytic* methods are correlational" (p. 87, emphasis in the original), and Knapp (1978) proffered mathematical proofs of this assertion. In the present paper the relationships of selected analytic procedures (t- test, Pearson correlation, multiple regression, ANOVA, MANOVA, and discriminant analysis) are examined and the correlational nature of such relationships are illustrated as special cases of canonical correlation analysis.

Demonstrating that Canonical Analysis Subsumes Other Statistical Procedures

Hypothetical data presented in Table 1 are used to illustrate the dynamics of canonical correlation analysis. The data consist of 24 cases concerning opinions (on a scale of 1-20) of contemporary issues on which Democrats and Republicans often differ. The predictor variables are PARTY (party affiliation of subjects) and AGE (of subjects). The criterion variables include LESSFED (less federal involvement), LESSWEL (less welfare spending), MOREDEF (more defense), MORESS (more social security), and CATMED (catastrophic medical program). Coding variables, also shown in the data set, are explained when comparing canonical correlation with factorial ANOVA and factorial MANOVA. For each of the six parametric methods used in the demonstration, a canonical correlation analysis is also computed. Appendix A containing the SAS commands for each procedure is provided for those who wish to replicate the analysis.

[INSERT TABLE 1 HERE]

Canonical Analysis Subsumes *t*-test

As novice researchers know, *t*-tests are used to determine if two groups differ statistically on a variable of interest. In this example, a *t*-test is used to ascertain whether Democrats and Republicans differed in their attitude toward LESSFED. The *t*-test results, reported in Table 2, indicate a statistically significant difference is found, t=-9.0418, p=.0001. When the difference between these two groups is determined using canonical analysis, the result is F=81.7548, p=.0001, which is also reported in Table 2. The lambda value presented in the table is Wilks's lambda, a value which ranges from zero to one and is used in this study for testing the significance of a canonical correlation. According to

Chacko (1986), the smaller lambda is -- that is, the closer to zero -- the greater the likelihood of statistical significance.

As can be seen in the table, a different distribution is used to report the results of each procedure; however, the calculated *p* values are identical. Further, values in the *F* distribution are the square of values in the *t* distribution. A double check of the results shows that the square of -9.0418 (the *t* value) equals 81.7548 (the *F* value obtained in the canonical analysis).

[Insert Table 2 here]

Canonical Subsumes Pearson Correlation

Pearson correlation is the statistic most commonly used to explore a relationship between two variables. A perfect correlation between two variables is $r = \pm 1$; when variables are perfectly uncorrelated the correlation is $r = 0$. In our example, a Pearson correlation is computed for the relationship between MOREDEF and MORESS, and PROC CANCORR for the canonical analysis. The results, reported in Table 3, show that the SAS command PROC CORR for the Pearson correlation computed a correlation coefficient of $r = -.4412$ ($p = .0309$); PROC CANCORR, the SAS command for canonical analysis, produced the canonical correlation coefficient, $Rc = .4412$ ($p = .0309$). A canonical correlation coefficient cannot be negative; however, the magnitude of the two coefficients is identical, as is the calculated probability.

[Insert Table 3]

Canonical Subsumes Multiple Regression

Hinkle, Wiersma, and Jurs (1988) define regression as "the process of predicting or estimating scores on a *Y* variable based on knowledge of scores on an *X* variable" (p. 441). Multiple regression simply expands that concept by using several variables to predict scores on the criterion variable. In this example, MORESS is the criterion variable and LESSFED, LESSWEL, and MOREDEF serve as the predictor variables. The results of the multiple regression and the canonical analysis are presented in Table 4.

In this case we find that the squared multiple correlation coefficient (R^2) derived from the regression analysis is R^2=.2182 (F=1.861, df=3, 20, p=1688). The canonical analysis resulted in a squared canonical correlation coefficient of R_c^2=.2182 (F=1.8605, df=3, 20, p=.1688). The difference in the value of F is the result of rounding.

[Insert Table 4]

Canonical Subsumes Factorial ANOVA

Analysis of variance procedures such as ANOVA and MANOVA require that intervally scaled ta be reduced to nominal scale. (Kerlinger, 1986; Thompson, 1988). In the present study, the predictor variable PARTY is nominally scaled; however, AGE is intervally scaled. Therefore, the variable AGEPR (age prime), found in Table 1, was created to divide AGE values into a trichotomy, with subjects under 30 designated as 1, those 31-45 designated as 2, and those 46 and older designated as 3.

For analytic purposes, *a priori* orthogonal contrast coding is used. The coding variables and values are also found in Table 1.

Kerlinger and Pedhazur (1973, pp. 131-140) explain how numeric values are assigned in orthogonal contrasts; however, Thompson (1987) notes that "contrasts are uncorrelated or orthogonal when the contrasts each sum to zero and when the cross-products of each pair of contrasts all sum to zero also" (p. 8).

Orthogonal planned comparisons are superior to analysis of variance for two reasons. First, planned comparisons offer more power against committing a Type II error. In other words, it is more likely that the researcher who uses planned comparisons will find a statistically significant effect if it exists (see Thompson, 1987 for a discussion). Secondly, planned comparisons force the researcher to be more thoughtful in conducting research "since the number of planned comparisons that can be tested is limited by the number of degrees of freedom for an effect" (Thompson, 1987, p. 11).

Even though orthogonal planned comparisons and other *a priori* contrasts represent an improvement over converting interval data to nominal scale, canonical correlation is the preferred method of analysis because no information is discarded as occurred with AGE in this study. Sacrificing variance is avoided when the data are already nominal, as is seen in this example with PARTY (Thompson, 1985a). However, nominally scaled variables are infrequently used in social science research.

The number of contrast variables needed equals the degrees of freedom (Thompson, 1987). Thus, AGEPR, for which there are two degrees of freedom, is represented by the orthogonal contrasts C1 and C2, while PARTY, for which there is one degree of freedom is designated with the orthogonal contrast CPTY. Both CPTY1 and CPTY2 are crossproducts, in this case of CPTY by C1, and CPTY by C2, respectively.

Using the SAS command PROC ANOVA, a 3 x 2 factorial ANOVA was computed with AGEPR and PARTY as independent variables and MORESS as the dependent variable. Results are reported in Table 5 for the main effects of AGEPR (F=5.81, p=.0113) and PARTY (F=14.67, p=.0012), and for the PARTY by AGEPR interaction (F=2.75; p=.0906). The error effect for the full ANOVA model is .3614711, and can be computed by dividing the sum of squares error by the sum of squares total (172.00000 / 475.83333) (Campo, 1990).

[Insert Table 5]

To obtain comparative results with canonical analysis, a series of four calculations was run using the contrast coding variables and the SAS command PROC CANCORR. Model 1, the first canonical analysis, provides a full model analysis using all five contrast coding variables, C1, C2, CPTY, CPTY1, and CPTY2 as the predictor set. Model 2 analyzes the three coding variables CPTY, CPTY1, and CPTY2 as predictors, while Model 3 uses four contrast coding variables C1, C2, CPTY1, and CPTY2 as predictor variables, and Model 4, the fourth canonical analysis, uses the three contrast variables, C1, C2, and CPTY, as predictors.

Lambdas for these calculations are presented in Table 6. According to Thompson (1985), lambda is analogous to a sum of squares in an ANOVA, and is an estimate of an effect. Unlike the sum of squares, however, a small lambda that approaches zero is desirable, as noted previously. The relationship between lambda and sum of squares can be seen in Table 6 where the full model lambda (.3614711) is the same value obtained above by dividing the sum of squares error by the sum of squares total. According to Thompson (1988), the squared canonical correlation coefficient equals 1 - lambda (full model), in this case *Rc*=.6385289.

[Insert Table 6 here]

The next step in the analysis is to convert the canonical lambdas to separate omnibus ANOVA effects. This is done by dividing the full model lambda (Model 1) by the lambda value for each effect (Models 2, 3, and 4). To compute the ANOVA lambda for the AGEPR main effect, the Model 1 canonical lambda is divided by the Model 2 canonical lambda. Specifically, in order to obtain the ANOVA lambda for the AGEPR main effect, the full model lambda which used all five contrast coding variables (C1, C2, CPTY, CPTY1, CPTY2) is divided by the Model 2 lambda which used the contrast coding variables for the PARTY main effect and two-way interaction effect (CPTY, CPTY1, CPTY2). Using values from Table 7, this conversion for AGEPR is: (.36147110/.59492119 = .607595). To derive the *F* value, another conversion is needed using the formula [(1-lambda)/lambda] * (df error/df effect) = *F*. This conversion is reported in Table 8. For the AGEPR main effect, the conversion is [(1-.607595)/.607595] * (18/2) = 5.8125. Notice that the F calculations in Table 8 are the same as the ANOVA F calculations in Table 5.

[Insert Table 8]

Canonical Subsumes MANOVA

A 2 X 3 factorial MANOVA using AGEPR and PARTY as the independent variables, and LESSFED and MOREDEF as the dependent variables was calculated. Table 9 reports values for lambda, *F*, and the associated probabilities. As with the ANOVA calculations presented above, four canonical analyses were computed using the contrast coding variables, C1, C2, CPTY, CPTY1, and CPTY2, to test for the main effects of AGEPR, PARTY, and the two-way interaction. Lambda values for the four canonical computations are presented in Table 10. Table 11 presents the conversion of the canonical lambdas into MANOVA lambdas. Note that the conversions result in values for lambda that are within rounding error of the Table 9 MANOVA lambdas.

[Insert Tables 9, 10, & 11 here]

Canonical Subsumes Discriminant Analysis

According to Afifi and Clark (1984), "discriminant analysis techniques are used to classify individuals into one or two or more groups" (p. 247). Discriminant analysis is also useful for descriptive purposes, identifying variables "which...contribute to making the classification" (Afifi & Clark, 1984, p. 247). Most often, discriminant analys s is used for classification purposes.

Discriminant analysis is similar to canonical analysis in that functions are extracted which represent linear combinations of the data. However, for discriminant analysis, the number of functions derived equals the smaller of (1) the number of groups minus one, or (2) the number of predictor variables. In this analysis, one predictor variable, PARTY, is used to predict scores on MORESS and CATMED. Thus, only one discriminant function is extracted from the data. The SAS command PROC CANDISC computed a discriminant analysis resulting in one significant function (F=5.9130, p=.0092), presented in Table 12. Also computed was a squared canonical correlation coefficient of Rc=.360265.

To calculate the corresponding canonical correlation analysis, the contrast coding variable CPTY was used as the predictor. The results of the canonical analysis, also presented in Table 12, are identical to the findings that emerged from the discriminant analysis.

Conclusion

The purpose of this study was to illustrate that canonical correlation analysis is the most general case of the general linear model. This attribute of canonical analysis is more important for heuristic value than for practical application, as can be determined from the ANOVA and MANOVA examples. However, this heuristic value should not be disregarded. Understanding that there is a common correlational basis for all parametric procedures may prompt researchers to be more selective in their choice of analytic techniques, particularly when considering OVA methods, such as ANOVA and MANOVA which discard variance.

Indeed, had all the variability of the data been subjected to a canonical analysis rather than using contrast coding, different conclusions may have resulted. The coding used in this example served the useful purpose of illustrating that OVA procedures are subsumed by canonical analysis. In a practical research context, however, understanding that results may differ, depending upon whether variance is discarded in OVA techniques or taken advantage of in more general methods like canonical analysis, takes on great importance.

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Table 1

DATA SET FOR CCA SUBSUMES ALL

CASE	PARTY	AGE	MORESS	CATMED	LESSFED	LESSWEL	MOREDEF	AGEPR	C1	C2	CPTY	CPTY1	CPTY2
1	1	35	20	19	13	15	12	2	0	2	-1	0	-2
2	1	25	17	19	16	12	14	1	-1	-1	-1	1	1
3	1	30	13	12	10	10	11	1	-1	-1	-1	1	1
4	1	29	13	14	9	10	10	1	-1	-1	-1	1	1
5	1	58	17	20	8	10	9	3	1	-1	-1	-1	1
6	1	25	14	15	7	8	8	1	-1	-1	-1	1	1
7	2	19	6	7	20	20	19	1	-1	-1	1	-1	-1
8	2	34	15	13	19	20	19	2	0	2	1	0	2
9	2	33	16	15	18	20	19	2	0	2	1	0	2
10	2	33	6	4	17	18	17	2	0	2	1	0	2
11	2	24	6	5	15	15	15	1	-1	-1	1	-1	-1
12	1	42	11	10	8	7	8	2	0	2	-1	0	-2
13	1	40	9	10	8	8	6	2	0	2	-1	0	-2
14	2	43	15	14	18	18	17	2	0	2	1	0	2
15	1	50	18	18	10	10	10	3	1	-1	-1	-1	1
16	2	60	15	15	17	18	17	3	1	-1	1	1	-1
17	2	26	6	6	20	19	19	1	-1	-1	1	-1	-1
18	2	20	5	4	19	19	20	1	-1	-1	1	-1	-1
19	2	52	14	14	16	15	16	3	1	-1	1	1	-1
20	1	50	18	18	10	10	9	3	1	-1	-1	-1	1
21	2	60	12	11	15	14	15	3	1	-1	1	1	-1
22	1	55	17	18	9	10	9	3	1	-1	-1	-1	1
23	1	39	17	15	8	7	5	2	0	2	-1	0	-2
24	2	53	10	12	18	20	18	3	1	-1	1	1	-1

Table 2
Canonical Subsumes *t*-tests
(PARTY by LESSFED)

t-test analysis		Canonical analysis	
Mean for Democs.	9.67	Squared Rc	.7880
SD	2.54	Rc	.8877
Mean for Repubs.	17.67	lambda	.2120
SD	1.72		
t	-9.0418	F	81.7548
df	22	df	1, 22
p calc	.0001	p calc	.0001

Table 3
Canonical Analysis Subsumes Pearson Correlation
(MOREDEF with MORESS)

Pearson Correlation		Canonical Analysis	
		Squared Rc	.1947
r	-.4412	Rc	.4412
		lambda	.8053
		F	5.3180
		df	1, 22
p calc	.0309	p calc	.0309

NOTE: *Rc* cannot be negative.

Table 4
Canonical Subsumes Multiple Regression
(MORESS with LESSFED, LESSWEL, & MOREDEF)

Multiple Regression		Canonical Analysis	
R Squared	.2182	Squared Rc	.2182
		Rc	.4671
		lambda	.7818
F	1.861	*F*	1.8605
df	3, 20	df	3, 20
p calc	.1688	*p* calc	.1688

Table 5
Factorial ANOVA
(AGEPR and PARTY by MORESS)

Source	SOS	df	MS (SOS/df)	*F*	p calc
AGEPR	111.0833	2	55.54167	5.81	.0113
PARTY	140.1667	1	140.16667	14.67	.0012
AGEPR*PARTY	52.58333	2	26.29167	2.75	.0906
ERROR	172.00000	18	9.55556		
TOTAL	475.83333	23			

Table 6
Canonical Analyses Using Four Models

Model	Predictors of MORESS	lambda
1	C1, C2, CPTY, CPTY1 CPTY2	.36147110
2	CPTY, CPTY1, CPTY2	.59492119
3	C1, C2, CPTY1, CPTY2	.65604203
4	C1, C2, CPTY	.47197898

Table 7
Conversion of Lambdas into Ratios for Each Effect

Effect	Models	Conversion	Result
AGEPR	1/2	.36147110/.59492119	.607595
PARTY	1/3	.36147110/.65604203	.550989
AGEPR*PARTY	1/4	.36147110/.47197898	.765863

Table 8
Conversion of Results to ANOVA *F*'s

Source	[(1-lambda)/lambda]	*	(df error/ df effect)	=	*F* calc
AGEPR	[(1-.607595)/.607595]	*	(18/2)		
	.6458332	*	9		5.81250
PARTY	[(1-.550989/.550989]	*	(18/1)		
	.8149183	*	18		14.66860
AGEPR*	[(1-.765863/.765863]	*	(18/2)		
PARTY	.3057166	*	9		2.75145

Table 9
Factorial MANOVA
AGE and PARTY by LESSFED and MOREDEF

Source	Lambda	df	*F*	*p* calc
AGEPR	.72382289	4/34	1.4909	.2267
PARTY	.14656153	2/17	49.4961	.0001
AGEPR*PARTY	.71855157	4/34	1.5274	.2162

Table 10
Canonical Analysis Using Four Models

Model	Predictors of LESSFED & MOREDEF	lambda
1	C1, C2, CPTY, CPTY1 CPTY2	.10314121
2	CPTY, CPTY1, CPTY2	.14249510
3	C1, C2, CPTY1, CPTY2	.70374003
4	C1, C2, CPTY	.14354045

Table 11
Conversion to MANOVA Lambdas

Effect	Models	Conversion	Results
AGEPR	1/2	.10314121/.14249510	.723823
PARTY	1/3	.10314121/.70374003	.146562
AGEPR*PARTY	1/4	.10314121/.14354045	.718552

Table 12
Canonical Subsumes Discriminant Analysis
(PARTY with MORESS and CATMED)

Discriminant Analysis		Canonical Analysis	
Function I		Function I	
Squared *Rc*	.360265	Squared *Rc*	.360265
Rc	.600220	*Rc*	.600220
lambda	.639735	lambda	.639735
F	5.9130	*F*	5.9130
df	2, 21	df	2, 21
p calc	.0092	*p* calc	.0092

APPENDIX A

SAS COMMANDS FOR CANONICAL SUBSUMES ALL PARAMETRIC TESTS

```
OPTIONS LS=80;
TITLE 'CCA SUBSUMES ALL FOR SERA 1993';
DATA ONE;
INFILE KADI;
INPUT CASE 1-2 PARTY 4-5 AGE 7-8 MORESS 10-11 CATMED 13-14
LESSFED 16-17
LESSWEL 19-20 MOREDEF 22-23 AGEPR 25-26 C1 28-29 C2 31-32 CPTY
34-35 CPTY1 37-38
CPTY2 40-41;
PROC PRINT;
TITLE 'DATA SET FOR CCA SUBSUMES ALL';
PROC MEANS;
 VAR MORESS CATMED LESSFED LESSWEL MOREDEF;
PROC SORT; BY PARTY;
PROC MEANS;
 VAR MORESS CATMED LESSFED LESSWEL MOREDEF;
 BY PARTY;
TITLE 'MEANS OF VARIABLES';
PROC TTEST;
 CLASS PARTY; VAR LESSFED;
TITLE 'CCA SUBSUMES TTEST';
PROC CANCORR;
 VAR LESSFED;
 WITH PARTY;
TITLE 'CCA SUBSUMES TTEST';
PROC CORR;
 VAR MOREDEF MORESS;
TITLE 'CCA SUBSUMES CORRELATION';
PROC CANCORR;
 VAR MORESS;
 WITH MOREDEF;
TITLE 'CCA SUBSUMES CORRELATION';
PROC REG;
 MODEL AGE=LESSFED LESSWEL MOREDEF;
TITLE 'CCA SUBSUMES REGRESSION';
PROC CANCORR ALL;
 VAR AGE;
 WITH LESSFED LESSWEL MOREDEF;
TITLE 'CCA SUBSUMES REGRESSION';
 PROC ANOVA;
 CLASS AGEPR PARTY;
 MODEL MORESS=AGEPR PARTY AGEPR*PARTY;
TITLE 'CCA SUBSUMES FACTORIAL ANOVA';
PROC CANCORR;
 VAR MORESS; WITH C1 C2 CPTY CPTY1 CPTY2;
PROC CANCORR;
 VAR MORESS; WITH CPTY CPTY1 CPTY2;
PROC CANCORR;
```

```
 VAR MORESS; WITH C1 C2 CPTY1 CPTY2;
PROC CANCORR; VAR MORESS; WITH C1 C2 CPTY;
TITLE 'CCA SUBSUMES FACTORIAL ANOVA';
PROC ANOVA;
 CLASS AGEPR PARTY;
 MODEL LESSFED MOREDEF=AGEPR PARTY AGEPR*PARTY;
MANOVA H=_ALL_;
TITLE 'CCA SUBSUMES FACTORIAL MANOVA';
PROC CANCORR;
 VAR LESSFED MOREDEF; WITH C1 C2 CPTY CPTY1 CPTY2;
PROC CANCORR;
 VAR LESSFED MOREDEF; WITH CPTY CPTY1 CPTY2;
PROC CANCORR;
 VAR LESSFED MOREDEF; WITH C1 C2 CPTY1 CPTY2;
PROC CANCORR;
 VAR LESSFED MOREDEF; WITH C1 C2 CPTY;
TITLE 'CCA SUBSUMES FACTORIAL MANOVA';
PROC CANDISC ALL;
 VAR MORESS CATMED; CLASS PARTY;
TITLE 'CCA SUBSUMES DISCR ANAL';
PROC CANCORR ALL;
 VAR MORESS CATMED; WITH CPTY;
TITLE 'CCA SUBSUMES DISCR ANAL';
```